geometrical changes tend to decrease performance. Single jet results show that upstream boattailing can reduce the drag of a cylindrical pod by 50% or more due to favorable interference.

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Approximate Analysis of a Flat, Circular Parachute in Steady Descent

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A theory is presented for the stress analysis of a flat, circular parachute in steady, vertical descent. Unlike previous treatments of the problem, this theory does not assume that the shape is known. Instead, the theory derives relations between the pressure distribution in the opened condition and the shape, drag and stresses in lines and fabric. The theory results in a nonlinear third order system of ordinary differential equations with boundary conditions at both vent and skirt. This system was solved by a computer program based on the Runge-Kutta method of numerical integration. The results are in fairly good agreement with measurements on parachutes. The computer program can be used for studies of effects of design changes on shape, drag and stress, and the results of a small study of this sort are included.

Nomenclature

A_0,A_1,A_2,A_3 = constants in canopy pressure distribution
C_{D0} = drag coefficient based on flat circular area
C_p = coefficient of net pressure
C_c,C_f = stress coefficients in cords, fabric
D = drag of canopy
D_p,D_0 = diameters in opened and flat circular states
E_c, E_f, E_L = elastic moduli of cords, fabric, load lines
f = $1 + (N_c/E_c)$
G = number of gores
J = dimensionless load line length
L_0,L = length of load lines in undeformed and deformed states
N_c,N_f = tension force in cords, fabric
N_θ,N_n = force resultants on cords in circumferential and normal directions
n_s,n_f = dimensionless force in cord, fabric
n_c = dimensionless circumferential force resultant on cords
p = net outward pressure on gore fabric
q = dimensionless pressure
R = radius of particle in flat circular state
R_i,R_0 = vent and skirt radii in flat circular state
r = radius of cord in opened state
r_{AB} = circumferential radius of curvature of gore
s = arc length of cord
U_f,U_L = dimensionless elastic constants
V = vertical velocity
W = dimensionless drag
x = dimensionless radius in flat, circular state
x_0 = dimensionless skirt radius in flat circular state
y = dimensionless radius in opened state
Z,Z_g = axial cylindrical coordinate of cords and gore centerline
z,z_g = dimensionless axial coordinate of cords and gore centerline
α = included angle of gore
β = edge angle of gore bulge
$\gamma_s,\gamma_f,\gamma_L$ = strains in cord, fabric, and load lines
δ = depth of bulge
Δ = dimensionless depth of bulge
θ = angle between load line and axis
Λ = load line length ratio = $L_0/D_0 = JR_i/(2R_0)$
ρ = mass density of air
σ' = contact length between adjacent gores
σ = dimensionless contact length between adjacent gores
τ = dimensionless function measuring contact of adjacent gores
ϕ = angle between cord direction and horizontal

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1. Introduction

In recent years repeated attempts have been made to analyze the behavior of parachutes and devise formulas suitable for their design. Much progress has been made, but the state of knowledge is still not wholly satisfactory. The best analyses currently available, Heinrich and Jamison[1] and Topping, Marketos, and Costakos[2] rely on knowing the deformed shape as well as the pressure distribution before the stresses are analyzed even in steady descent. Present knowledge about large deflections of structures suggests that it should be possible to calculate the stresses and deformed shape concurrently, although some assumptions must still be made about the pressure distribution in the deformed state. We shall undertake to do this in the present paper.

To be specific, we shall analyze a flat, circular canopy in steady, vertical descent by using the ideas of large-deforma-

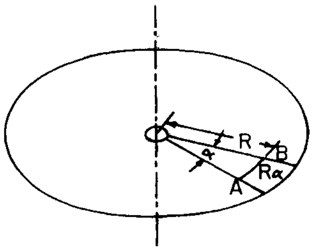

Fig. 1 Undeformed parachute gore.

tion, elastic theory. Our aim is to develop a procedure for estimating the stresses and shape of the canopy when the pressure distribution is known. Ideally, we want the procedure to be the simplest possible one that will predict the canopy behavior with reasonable accuracy, and so we make a large number of simplifying assumptions, of which the most important are the following: 1) the fabric and lines all have zero bending stiffness; 2) the strain in the fabric and lines is small everywhere, even though the deflections may be large; and 3) the deformations and stresses are the same in all the gores and in all the lines. The remaining assumptions will be described when they are invoked.

The analysis, described in the following three sections, bears some resemblance to (but goes beyond) that of Heinrich and Jamison.[1] A computer program, described briefly in Sec. 5, was written to carry out the calculations demanded by the analysis. Section 6 presents the results of applying the computer program to the case of a C-9 parachute and a discussion is given in Sec. 7.

2. Analysis of the Fabric

Since all gores undergo the same stress and deformation (assumption 3), it suffices to discuss just one of them. The reference, or undeformed, state of this typical gore is taken as a flat sector of a circle (see Fig. 1).

We begin by examining the shape of this gore when the canopy is open and finding formulas for the strain in terms of the shape parameters. We consider the points lying on the circular arc AB (see Fig. 1) in the undeformed state and analyze what happens to them in the open or deformed condition. We assume that in the open condition the arc AB forms an approximately plane curve, the plane of the curve being roughly perpendicular to the cords (Fig. 2). Actually surfaces containing AB will always be slightly curved, but we often neglect this in what follows. Usually AB has the shape of a circular arc with radius r_{AB} (Fig. 3). However, sometimes adjacent gores are in contact over a length σ' as shown in Fig. 4. In the former case ($\sigma' = 0$) we obtain from the geometry of Figs. 2 and 3,

$$r_{AB} \sin\beta = (\tfrac{1}{2})r\alpha,\ \beta < \pi/2 \tag{1}$$

$$\delta = r_{AB}(1 - \cos\beta) = (\tfrac{1}{2})r\alpha \tan(\beta/2) \tag{2}$$

$$\text{Deformed length of } AB = 2\beta r_{AB} \tag{3}$$

and in the latter case ($\sigma' > 0$), Figs. 2 and 4 imply

$$r_{AB} = r\alpha/2 \tag{4}$$

$$\delta = (r\alpha/2) + \sigma' \tag{5}$$

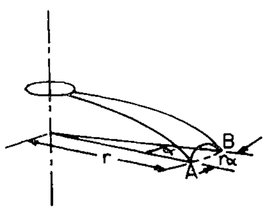

Fig. 2 Deformed parachute gore.

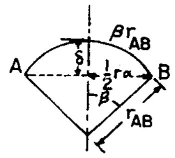

Fig. 3 Section of deformed gore normal to cords, $\sigma' = 0$.

$$\text{Deformed length of } AB = 2\sigma' + (\pi r\alpha/2) \tag{6}$$

$$\beta = \pi/2 \tag{7}$$

Since the original length of AB is $R\alpha$ (see Fig. 1), we find for the circumferential fabric strain

$$\gamma_f = (2\beta r_{AB}/R\alpha) - 1 \text{ if } \sigma' = 0 \tag{8}$$

$$= \{[2\sigma' + (\pi r\alpha/2)]/R\alpha\} - 1 \text{ if } \sigma' > 0 \tag{9}$$

Now we shall analyze the stress in the fabric and its effect on the cords. In doing so we assume that all meridional stresses are borne by the cords, the meridional fabric stress being everywhere zero. With this assumption, equilibrium of the gore requires

$$N_f = pr_{AB} \tag{10}$$

The forces exerted by the fabric on the cords per unit cord length, are shown in Fig. 5, whence we derive the equilibrium relations

$$N_\theta = N_f \cos\beta \tag{11}$$

$$N_n = 2N_f \sin\beta \tag{12}$$

Combining these with Eqs. (10, 1, 4, or 7) as appropriate we find

$$N_\theta = (\tfrac{1}{2})pr\alpha \cot\beta,\ N_n = pr\alpha \text{ for } \sigma' = 0 \tag{13}$$

$$N_\theta = 0,\ N_n = pr\alpha,\ N_f = pr\alpha/2 \text{ for } \sigma' > 0 \tag{14}$$

Finally we assume the fabric obeys a linear elastic law

$$N_f = E_f\gamma_f \tag{15}$$

This relation can be put into various useful forms. When $\sigma' = 0$, we may combine it with Eqs. (8, 1, 10, and 11) to obtain the relations

$$\sin\beta - (\beta r/R) + [pr\alpha/(2E_f)] = 0 \tag{16}$$

$$N_\theta/E_f = \cos\beta\{(r/R)\beta \csc\beta - 1\} \tag{17}$$

$$N_f/E_f = (r/R)\beta \csc\beta - 1 \tag{18}$$

When $\sigma' > 0$, (15) may be combined with Eqs. (9, 10, and 4) to get

$$\sigma' = (R\alpha/2)\{1 - (\pi/2)(r/R) + (\tfrac{1}{2})(pr\alpha/E_f)\} \tag{19}$$

We observe that, when $\sigma' = 0$, Eq. (19) implies the same relation that we obtain from Eq. (16) by setting $\beta = \pi/2$, as it should. This implies that we may distinguish between the cases $\sigma' > 0$ and $\sigma' = 0$ by means of the quantity

$$\tau = 1 - (\pi/2)(r/R) + (\tfrac{1}{2})(pr\alpha/E_f) \tag{20}$$

When $\tau \leq 0$, $\sigma' = 0$, and when $\tau > 0$, $\sigma' > 0$.

This completes the analysis of the fabric. The results of this analysis are embodied in Eqs. (16, 17, and 18) when $\tau \leq 0$ and Eqs. (7, 14, and 19) when $\tau > 0$. These results will later be combined with those obtained by analyzing the cords to form the complete theory.

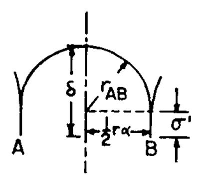

Fig. 4 Section of deformed gore normal to cords, $\sigma' > 0$.

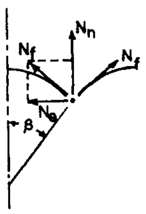

Fig. 5 Forces transmitted from gore fabric to cords.

3. Analysis of the Cords

We begin by analyzing the geometry and strain in the cords. From Fig. 6 we see that in the deformed state

$$dr/ds = \cos\phi \tag{21}$$

$$dZ/dr = \tan\phi \tag{22}$$

The cord element that has length ds in the deformed state is assumed to have length dR in the undeformed state, hence the strain in the cord is found with aid of Eq. (21) to be

$$\gamma_s = (ds/dR) - 1 = (dr/dR)\sec\phi - 1 \tag{23}$$

Next we write down the equations for equilibrium of forces acting on a cord element in the directions tangential and normal to the cords with the help of Fig. 6. These forces include those transmitted from the fabric to the cords.

$$dN_c/ds = 2N_\theta \sin(\alpha/2)\cos\phi \tag{24}$$

$$N_c\, d\phi/ds = N_n - 2N_\theta \sin(\alpha/2)\sin\phi \tag{25}$$

Finally, the cords are assumed to obey a linear elastic relation,

$$N_c = E_c\gamma_s \tag{26}$$

Several other useful relations may be derived from the previous. If we define

$$f = 1 + (N_c/E_c) \tag{27}$$

We obtain from Eqs. (23) and (26)

$$dR/ds = 1/f \tag{28}$$

$$dr/dR = f\cos\phi \tag{29}$$

If we use the result $N_n = pr\alpha$ and Eq. (27), we may put Eqs. (24) and (25) into the form

$$dN_c/dR = 2fN_\theta \sin(\alpha/2)\cos\phi \tag{30}$$

$$d\phi/dR = (f/N_c)\{pr\alpha - 2N_\theta \sin(\alpha/2)\sin\phi\{ \tag{31}$$

Also, we may combine Eqs. (22) and (28) to get

$$dZ/dR = f\sin\phi \tag{32}$$

This completes the analysis of the cords. Equations (27 and 29–32) will be used in the final version of this theory.

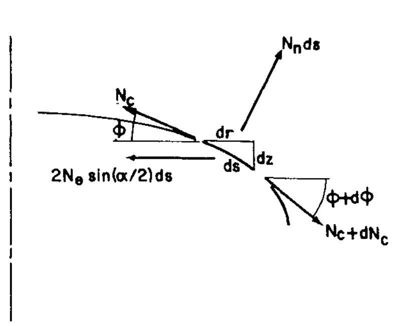

Fig. 6 Forces acting on an element of cord with length *ds*.

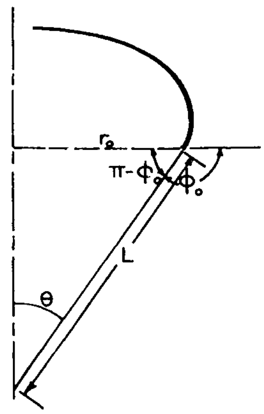

Fig. 7 Geometry of skirt and load lines.

4. Synthesis of the Theory

Equations (29–31) form a set of three nonlinear ordinary differential equations for the three quantities r, N_c, and ϕ as functions of R. N_θ, which occurs, as a parameter in (30) and (31), is an implicit function of r and R that can be found from the results of Sec. 3 without further differentiation. Thus, the entire system is of third order, and we expect that we shall need three boundary conditions to make the solution unique.

We shall impose two boundary conditions at the vent,

$$\phi(R_i) = 0 \tag{33}$$

$$r(R_i) = R_i\{1 + [N_c(R_i)/E_c]\} = R_i f(R_i) \tag{34}$$

The first of these stems from the fact that no vertical forces act on the vent. The second arises from the elastic law, Eq. (26), and the strain definition, Eq. (23), applied to the portion of the cord between the vent and the centerline. A third condition is obtained from Fig. 7 and is applied at the skirt,

$$r(R_0) - L\cos[\pi - \phi(R_0)] = r(R_0) + L\cos\phi(R_0) = 0 \tag{35}$$

We may express L in terms of the strain and undeformed length of the suspension lines,

$$L = L_0(1 + 2\gamma_L)^{1/2} \tag{36}$$

We assume that the suspension lines obey the elastic relation

$$N_c(R_0) = E_L\gamma_L \tag{37}$$

and therefore, combining Eqs. (35–37), we find

$$r(R_0) + L_0\{1 + [2N_c(R_0)/E_L]\}^{1/2}\cos\phi(R_0) = 0 \tag{38}$$

as our third boundary condition.

In applying this theory, the main problem is to solve the differential equation system Eqs. (29–31) with the three boundary conditions just stated. Assuming that this can be done, other information such as the drag and deformed shape can be found. The drag is given by

$$D = GN_c(R_0)\sin\phi(R_0) \tag{39}$$

The deformed shape is defined by the cord and gore centerline profiles. The cord profile is given parametrically by $r(R)$ and $Z(R)$, where $Z(R)$ is found by integrating Eq. (32),

$$Z(R) = \int_{R'=0}^{R} f(R')\sin\phi(R')dR' \tag{40}$$

The gore centerline profile, defined parametrically by $r_g(R)$ and $Z_g(R)$, is found from the geometrical relations (Fig. 8)

$$r_g(R) = r(R) + \delta(R)\sin\phi(R) \tag{41}$$

$$Z_g(R) = Z(R) - \delta(R)\cos\phi(R) \tag{42}$$

Thus, the shape and drag in the open condition can be determined if the differential equation system can be solved. The cord stress is found directly in solving the differential equation system, and the fabric stress is found by using Eq. (14) or (18). Thus, the theory gives us estimates of all the quantities we need, provided the differential equation system can be solved.

We shall now put the equations of the theory in dimensionless form by introducing the definitions

$$\begin{gathered} R = xR_i,\ r = yR_i,\ L_0 = JR_i,\ R_0 = x_0R_i \\ \sigma' = \sigma R_i,\ Z = zR_i,\ \delta = \Delta R_i,\ Z_g = z_gR_i \\ r_g = y_gR_i,\ N_c = E_cn_s,\ N_\theta = E_fn_c,\ N_f = E_fn_f \\ p = 2qE_f/R_i,\ D = E_cW,\ E_f = U_fE_c/E_f,\ E_L = U_LE_c \end{gathered} \tag{43}$$

Then in dimensionless form Eqs. (27, 20 and 29–31) are

$$f = 1 + n_s \tag{44}$$

$$\tau = 1 + q\alpha y - [\pi y/(2x)] \tag{45}$$

$$dy/dx = f\cos\phi \tag{46}$$

$$d\phi/dx = (2fU_f/n_s)\{q\alpha y - n_c\sin(\alpha/2)\sin\phi\} \tag{47}$$

$$dn_s/dx = 2fU_fn_c\sin(\alpha/2)\cos\phi \tag{48}$$

where n_c and various other gore parameters are found as functions of x by the following formulas: if $\tau > 0$, Eqs. (7) and (14) read

$$\begin{gathered} n_c = 0,\ n_f = q\alpha y \\ \beta = \pi/2,\ \sigma = x\alpha\tau/2 \end{gathered} \tag{49}$$

and, if $\tau \leq 0$, then from Eqs. (17) and (18)

$$n_c = \cos\beta\{(y/x)\beta\csc\beta - 1\} \tag{50}$$

$$n_f = (y/x)\beta\csc\beta - 1 \tag{51}$$

where β is determined by solving Eqs. (16),

$$\sin\beta - (\beta y/x) - q\alpha y = 0 \tag{52}$$

The edge conditions Eqs. (33, 34, and 38) are now

$$n_s(1) = y(1) - 1 \tag{53}$$

$$\phi(1) = 0 \tag{54}$$

$$y(x_0) + J\{1 + [2n_s(x_0)/U_L]\}^{1/2}\cos\phi(x_0) = 0 \tag{55}$$

The dimensionless load or drag, Eq. (39), becomes

$$W = Gn_s(x_0)\sin\phi(x_0) \tag{56}$$

and the functions specifying the deformed shape, Eqs. (40, 5, 2, 41, and 42), are now, respectively,

$$z(x) = \int_0^x f(x')\sin\phi(x')dx' \tag{57}$$

$$\Delta = \sigma + (y\alpha/2) \text{ if } \tau > 0 \tag{58}$$

$$\Delta = (y\alpha/2)\tan(\beta/2) \text{ if } \tau \leq 0 \tag{59}$$

$$y_g = y + \Delta\sin\phi \tag{60}$$

$$z_g = z - \Delta\cos\phi \tag{61}$$

We also define pressure, drag and stress coefficients by the following formulas

$$C_p = p/\{\rho V^2/2\} = \text{pressure coefficient} \tag{62}$$

$$C_{D0} = D/\{\rho V^2\pi R_0^2/2\} = \text{drag coefficient} \tag{63}$$

$$C_c = N_c/\{\rho V^2\pi R_0^2/2\} = \text{cord stress coefficient} \tag{64}$$

$$C_f = N_fR_0/\{\rho V^2\pi R_0^2/2\} = \text{fabric stress coefficient} \tag{65}$$

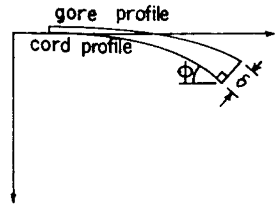

Fig. 8 Profiles of deformed cord and gore centerline.

Henceforth we shall assume that Eqs. (44) to (61) are the mathematical embodiment of our theory about parachutes in steady, vertical descent.

5. Computer Program

In applying the theory developed in Secs. 2–4, the main problem is to solve the differential equation system Eqs. (46–48) with the edge Eqs. (53–55). A general solution in closed form is not easily found, hence a computer program was written.

The program is based on the Runge-Kutta numerical integration scheme of second order and involves a trial-and-error method for meeting the edge conditions. The general procedure is well known, the only uncommon feature being the somewhat complicated calculation needed to find n_c and the other gore parameters at each point by means of Eqs. (49–52). The program assumes that the pressure coefficient function, $C_p(\phi)$, is of cubic polynominal form,

$$C_p(\phi) = A_0 + A_1\phi + A_2\phi^2 + A_3\phi^3 \tag{66}$$

and, hence $C_p(\phi)$ is determined by choosing the constants A_0, A_1, A_2, A_3.

We may summarize the program in the following way. It takes as input the quantities $R, R_0, G, J, E_c, E_L, \rho, V$, and A_0, A_1, A_2, A_3. As output it gives C_{D0} plus the following functions of x: $\phi, y, z, y_g, z_g, \beta, \sigma, C_c$, and C_f. The program is written so the user may choose to vary any two of the input quantities in order to find the effect of design changes on the performance characteristics of the canopy.

6. Examples of Results

This program was used to study the effects of different variables on the shape, drag, and stresses. For this purpose a basic set of parameter values was chosen, typical of the C-9 canopy: vent radius = 1.4 ft; ratio of skirt to vent radii = 10; number of gores = 28; suspension line length = 28 ft; elastic modulus of cords = 2×10^3 lb; elastic modulus of fabric = 2×10^3 lb/ft; and elastic modulus of load lines = 2×10^3 lb. The mass density of air was taken as $\rho = 2 \times 10^{-3}$ lb-sec/ft^4 and the drop velocity as $V = 20$ fps. The basic pressure distribution was taken as a constant function $C_p = 1.5$, defined by $A_0 = 1.5$, $A_1 = A_2 = A_3 = 0$. For this basic configuration, the following results were found: C_{D0} = Drag coefficient = 0.626; D_p = max. diam = $0.671D_0$; ϕ_0 = cord angle at skirt = 109°; or $\theta = \phi_0 - (\pi/2) = 19°$; where $D_0 = 2R_0$ is the flat circular diameter and θ is the angle between the suspension lines and the canopy axis. The deformed shape is shown in Fig. 9, and the cord and fabric stress coefficients are depicted in Fig. 10.

The theory was further tested by investigating the influence of various parameters on the shape, drag, and stresses. A large number of results were obtained, the most important of which are displayed in Tables 1 and 2. We may summarize the main features as follows:

1) Table 1 shows that D_p/D_0 and, hence, the shape, is not greatly affected by any of the parameter changes. The suspension line length ratio, Λ, has the greatest effect on D_p/D_0, but its effect is not especially large.

2) According to Table 1, $\Lambda = L_0/D_0$ is the only canopy parameter that affects the drag coefficient significantly.

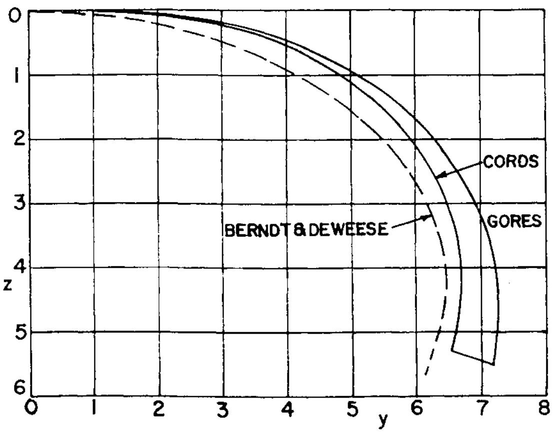

Fig. 9 Deformed shape for basic configuration.

The effects of G and V are small, and the elastic constants have negligible influence.

3) The angle θ of the load lines, which is the same as the angle of the cords at the skirt, is seen in Table 1 to be significantly affected only by changes in Λ.

4) We conclude from Table 1 that G is the only variable that greatly influences $C_{c\max}$ although Λ has some effect. In contrast $C_{f\max}$ is perceptibly affected by changes in all the canopy parameters.

5) The cord stress is least at the vent and increases to a maximum at the maximum diameter, which is usually at $x = 9$, i.e., near the skirt. Hence, the familiar formula $C_c = C_{D0}/G \cos\theta$, which is exactly true only for $C_c(x_0)$, is also approximately true for the maximum value of C_c.

6) The fabric stress is greatest at or near the vent and decreases as we go toward the skirt. The exact location of the maximum fabric stress depends on E_f as well as the pressure distribution.

7) From Table 2, we see that for constant pressure distributions, changing the pressure scarcely affects the shape and causes merely proportional changes in the stresses. If the pressure increased linearly in ϕ from the vent to the skirt, all quantities are affected. We shall comment on these results in the following section.

7. Discussion

The theory developed in this paper is apparently the first in which the shape and stresses are calculated simultaneously. To judge the theory we must, of course, compare it with experimental results. In making this comparison it is first necessary to understand clearly what the theory does.

If we are given a flat, circular canopy with known geometrical and physical properties (respectively embodied in R_i, R_0, L_0, G, and E_f, E_c, and E_L), dropping vertically at known velocity V through air with known mass density ρ, the analysis furnishes us with the relations between the pressure distribution on one hand and the shape, stresses and drag on the other. If the pressure distribution is known, we can solve these relations (by means of the computer program) to find the quantities of engineering interest. These quantities will depend in general upon the pressure distribution.

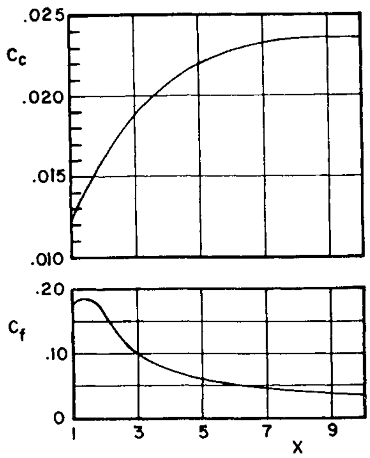

Fig. 10 Cord and fabric stress coefficients for basic configuration.

Table 1 Effects of varying design parameters

Parameter	D_p/D_0	C_{D_0}	Max C_c	Max C_f	$\theta = \phi_0 - (\pi/2)$
E_c					
1000	0.673	0.627	0.0237	0.264	19°
2000[a]	0.671	0.626	0.0237	0.185	19°
3000	0.671	0.625	0.0236	0.155	19°
E_f					
400	0.672	0.626	0.0237	0.087	19°
1200	0.672	0.626	0.0237	0.140	19°
2000[a]	0.671	0.626	0.0237	0.185	19°
G					
16	0.665	0.614	0.0406	0.219	19°
28[a]	0.671	0.626	0.0237	0.185	19°
40	0.674	0.630	0.0167	0.155	19°
Λ					
0.7	0.645	0.549	0.0218	0.175	26°
1.0[a]	0.671	0.626	0.0237	0.185	19°
1.3	0.687	0.671	0.0248	0.191	15°
V					
10	0.670	0.623	0.0236	0.210	19°
20[a]	0.671	0.626	0.0237	0.185	19°
30	0.673	0.629	0.0238	0.176	19°

[a] Denotes value of variable for the basic configuration.

An ideal experimental check on the theory would require that simultaneous measurements of pressure distribution, stresses, shape, and drag be made on a canopy that is dropping vertically. Conditions for such a test may be difficult or impossible to realize in practice, and it appears that no experiments yet made will permit such a complete check on the theory.

However, many experiments have been made that, while incomplete in some respect, give us the information for a partial check on the theory. For example, Berndt and Deweese[3] made measurements on a C-9 canopy in towed flight and obtained the results $C_{D0} = 0.65$, $D_p/D_0 = 0.648$, $\theta = 17°$ compared with the present predictions, $C_{D0} = 0.626$, $D_p/D_0 = 0.671$, $\theta = 19°$. Their measured shape is plotted in Fig. 9. The general agreement is good although the theoretical shape is slightly wider and shallower than the experimental shape. We may conclude from this that the present theory is not grossly in error. However, it is necessary to remark that, if we had assumed a different pressure distribution in the calculations, we could have arrived at quite different values for C_{D0}, as is evident from Table 2.

The Parachute Handbook[4] gives the general estimate $D_p/D_0 = 0.7$, which compares reasonably well with the

Table 2 Effects of changing the pressure distribution

Constant C_p	D_p/D_0	C_{D_0}	Max C_c	Max C_f	θ
1.2	0.671	0.500	0.0189	0.150	19°
1.5[a]	0.671	0.626	0.0237	0.185	19°
1.8	0.672	0.751	0.0284	0.220	19°
Linearly varying C_p[b]					
$C_p(x_0)$					
1.500[a]	0.671	0.626	0.0237	0.185	19°
1.882	0.689	0.707	0.0268	0.202	20°
2.268	0.702	0.785	0.0299	0.218	20°

[a] Denotes value of variable for the basic configuration.
[b] In the case labelled "linearly varying C_p," C_p varies linearly in ϕ between C_p (1) = 1.5 (at vent) and $C_p(x_0)$ at the skirt.

present calculations, $D_p/D_0 = 0.67$ for the cords and $D_p/D_0 = 0.73$ for the gore centerlines. Moreover, the observed generality of this estimate tends to confirm the theory, which predicts that the shape is insensitive to changes in most of the parameters.

The Parachute Handbook[4] shows many experimental results in which C_{D0} is seen to decrease with an increase in V. This does not agree, at first sight, with our results, Table 1, which show that C_{D0} increases very slowly with an increase in V. The discrepancy is probably due to one or both of two causes:

1) The canopy may not be descending vertically in the tests. The theory assumes vertical descent and may give inaccurate results if the canopy oscillates or glides laterally.

2) The pressure distribution may change as the velocity changes. In particular, increasing velocity will cause increase of the tensile stresses and strains in the fabric, and this will lead to greater porosity of the fabric. The increasing porosity will permit increasing air flow from the high-pressure to the low-pressure side of the fabric. Thus, the pressure difference will not increase as fast as the impact pressure, or, in other words, the pressure-difference coefficient C_p will decrease. This will cause C_{D0} to decrease.

In general we may say that the present theory represents a first, admittedly somewhat crude, attempt to analyze parachutes without assuming the deformed shape and without arriving at an impossibly complicated analysis. The results are not perfect but are good enough to suggest that the general procedure is a workable one which can be a starting point for future efforts.

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